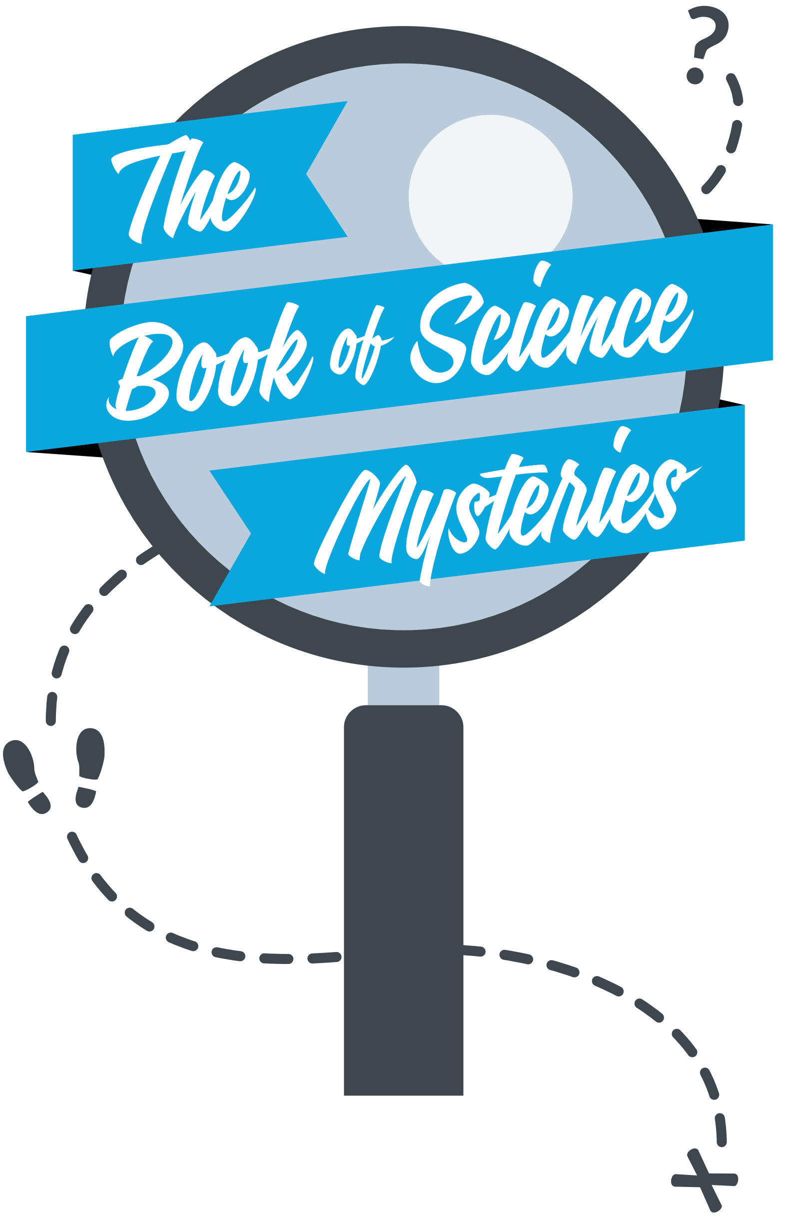
TEMI



The card colours command

CLASSROOM SCIENCE ACTIVITY TO
SUPPORT STUDENT ENQUIRY-BASED LEARNING

This classroom-tested teaching plan uses the four innovations of the TEMI project, as detailed in the Teaching the TEMI Way (TEMI, 2015).

You should read this companion book to get the most from your teaching. The **TEMI** techniques used in this teaching plan are: **1)** productive science mysteries, **2)** the **5E model** for engaged learning, **3)** the use of presentation skills to engage your students, and **4)** the apprenticeship model for learning through gradual release of responsibility. You might also wish to use the hypothesiser lifeline sheet (available on the **TEMI** website) to help your students document their ideas and discoveries as they work.

To know more about **TEMI** and find more resources **www.teachingmysteries.eu**

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teachingmysteries.eu

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The card colours command

What's the mystery?

A shuffled pack of cards seems to be under the control of your voice commands, as you can predict the number of red and black cards in two piles that are lying face down on the desk.

DOMAIN(S)

Mathematics.

SUBDOMAIN KEYWORDS

Algebra, equation, mathematical model, mathematical proof.

AGE GROUP

15 to **16** years old.

EXPECTED TIME FOR THE MYSTERY

Approximate time for teacher preparation:
One hour for practising the trick and understanding the algebra used in the solution.

Approximate time in classroom:
One hour in classroom with the students as they explore the trick and mathematics..

SAFETY/SUPERVISION

None, just ensure that a full deck of 52 cards is used.

Disclaimer: the authors of this teaching material will not be held responsible for any injury or damage to persons or properties that might occur in its use.

PREPARATION AND LIST OF MATERIALS

» A standard pack of 52 playing cards with no jokers.

» Pencil and paper to record information in experiments.

LEARNING OBJECTIVES

Student will learn about the basics of algebra, such as variables, constraints, and equation substitution.

Guidance notes for teachers

THE 5E MODEL

Engage

CAPTURE STUDENTS' ATTENTION

Say you will show your ability to control playing cards with the power of your voice!

Shuffle the cards and turn over the top card; if it's red, put it face up and deal another card face down while declaring that it is red. This pile is your red pile. If the top card is black, put it on the other side and deal another card face down while saying that it is black: this is your black pile. Go through the entire pack this way.

The piles on the table will look like this:

» Red cards face up.
» Face-down card pile called the red pile.

» Black cards face up.
» Face-down card pile called the black pile.

Though the deck is shuffled, your "word commands" ensure that there are the same number of red cards in the red pile as there are black cards in the black pile.

Explore

COLLECT DATA FROM EXPERIMENTS

Collect all the cards and shuffle again: then do a series of experiments exploring this phenomenon. You can have the students do these experiments in groups.

1. What happens if you say different words when dealing the facedown cards? Or don't say anything? Does it make any difference?
2. What happens if you just deal the cards randomly into four piles? Is there any pattern to the places where you find the red and black cards?
3. What happens if you shuffle the undealt cards in your hands as you go along?
4. What happens when you try different types of shuffles or get a friend to shuffle at the beginning?

Explain

WHAT'S THE SCIENCE BEHIND THE MYSTERY?

The trick is based on algebra and is a nice example of how to introduce the idea that algebra allows us to model a system; in this case, the system is how the cards end up in the piles.

Call the number of cards in the red pile R0: be sure to just use a letter instead of the actual number, as the actual number will be different each time you do the trick. Call the number of cards in the face up black pile B0, again just using a letter to let you be general about the number. What other piles on the table have R0 and B0 number of cards in them? As the deck is shuffled, you can't know the exact numbers of red and black cards in the piles, but again you can use letters: in this way, you can write down some facts about the numbers. In the red pile, let's say there are R1 red cards and B1 black cards: what do R1 and B1 need to add up to? In the black pile, let's say there are R2 red and B2 black: what do R2 and B2 need to sum to?

The number and the colours of cards on the table are represented algebraically as follows.

» **R0** (Red cards face up)
» **R1+B1** (Face-down card pile called the red pile)

» **B0** (Black cards face up)
» **R2+B2** (Face-down card pile called the black pile)

We have the equation **R0+R1+R2=26**, meaning that the total number of red cards on the table is 26 (half a pack of 52 cards). What can you say about **B0+B1+B2**: what's it equal to?

If $R1+B1= R0$, the number of face-down red and black cards in the red pile is equal to the number of face-up red cards in the pile above it: is there a way you can use this fact and the fact that $R2+B2=B0$ to show that R1 will always equal B2 for the trick if you follow the instructions?

Answer:
$R0 + R1 + R2 = 26$ (call this equation (1)).
$B0 + B1 + B2 = 26$ (call this equation (2)).
$R0 = R1 + B1$ (call this equation (3)).
$B0 = R2 + B2$ (call this equation (4)).

So if we substitute equation (3) into equation (1) and eliminate R1, we get:
$(R1 + B1) + R1 + R2 = 26$ (call this equation (5)).

Similarly, if we substitute equation (4) into equation (2) and eliminate B0, we get:
$(R2 + B2) + B1 + B2 = 26$ (call this equation (6)).

Combining equations (5) and (6), as both add up to 26, we get:
$(R1 + B1) + R1 + R2 = (R2 + B2) + B1 + B2$

Collecting similar terms gives us:
$2R1+B1+R2=2B2+R2+B1$

We can subtract R2 and B1 from each side of the equation. This leaves:
$2xR1 = 2xB2$

We can divide both sides by two, giving us $R1=B2$. This means that for every full pack of cards dealt with the correct procedure, the number of red cards in the face-down red pile always equals the number of black cards in the face-down black pile, which is exactly what your prediction is.

Extend

WHAT OTHER RELATED AREAS CAN BE EXPLORED?

In this trick, you have created what scientists call a **mathematical model.** By using letters to represent the number of cards, we can understand what's happening and be sure of the situations where the trick will always work. We can also use it to predict how things will work if we make changes. For example, you may be able to come up with different presentations or perhaps different predictions of the outcome if you change the number of cards in the pack.

The letters that represent numbers that can change are called variables because they can vary; here, B1 and R2 are variables. Constraints are things that fix the way the model acts; for example, here we have constraints that $R0+R1+R2=B0+B1+B2=26$ (we are using a full deck) and $R0=R1+B1$ (we deal a card face down under every face-up red card). **Variables and constraints** combine to give us powerful mathematical models in science and engineering; for example, they are very useful for checking if a bridge will carry traffic or making software do what you want.

Evaluate

CHECK THE LEVEL OF STUDENT SCIENTIFIC UNDERSTANDING

Set a few questions on the basics of algebraic manipulation.

E.g. If $A=B+C$ and $D=B$, then what can we say that's always true? Answer: $A=C+D$

Set the students a challenge. If you wanted to perform the trick so that there are equal numbers of red and black cards in the face-down piles, then the result would be exactly one more red card than black cards. How could you do this?

Answer: secretly remove two black cards before you start the trick.

THE 5E MODEL

Showmanship

TIPS ON HOW TO TEACH AND PRESENT THIS MYSTERY

This is a nice direct magic trick and you should present it as such; so long as the instructions are followed, it will work every time. This means you can try swapping the same number of cards from one face-down pile to the other, saying "you got those ones wrong". Develop your own style of presentation and enjoy the looks of astonishment.

GRR

TEACHING SKILLS USING GRADUAL RELEASE OF RESPONSIBILITY

Setting up the mystery: tell the class that they can determine how the cards fall by words alone.

Demonstrated enquiry (level 0): teacher-as-model. You show how to carry out an enquiry process, which the students then copy; for example, does the shuffling of the cards as you deal the piles make a difference? Explain your hypothesis and tests by "talking aloud". Students record their thinking onto their hypothesiser lifeline worksheet.

Structured enquiry (level 1): "we do it". Students use their hypothesiser lifeline sheet to record their own alternative ideas about why colours separate and to record their tests and conclusions regarding these other possible explanations.

Solving the mystery: students are led towards the explanation by using ideas about the use of algebra and abstraction to show why the trick works.

Resources

Videos showing this trick, as well as others, being performed and explained using a different "body language reading" presentation can be found at:
www.mathematicalmagic.com

The application of magic in teaching computer science algorithms and free books to download can be found at:
www.cs4fn.org/magic

The card colours command

STUDENT WORKSHEET

Can your thoughts and words control the colours of the unseen cards you deal when playing with a normal pack of cards?

Engage

WHAT'S INTERESTING?

Task: With a pack of cards in your hands, follow the teacher's instructions to create two random piles of face-down cards. Although you shuffled and dealt randomly, your teacher can magically predict the numbers of cards of each colour.

Explore

WHAT'S HAPPENING?

Task: Follow the teacher's instructions to perform the trick. The piles on the table will look like this:

» Red cards face up.
» Face-down card pile called the red pile.

» Black cards face up.
» Face-down card pile called the black pile.

What happens if you say different words when dealing the face-down cards? Or don't say anything? Does it make any difference?

What happens if you just deal the cards randomly into four piles? Is there any pattern to the places where you find the red and black cards?

What happens if you shuffle the undealt cards in your hands as you go along?

What happens when you try different types of shuffles or get a friend to shuffle at the beginning?

What happens if you just use ten red cards and ten black cards and follow the instructions?

Explain

WHAT'S CAUSING IT?

Task 1: Think about all the things that can change in this trick, like the number of cards in each pile. These are called variables and can be represented by letters rather than numbers.

The things that are always the same in this trick are called constraints. What can you say about the total number of red cards in all the piles? What about the number of cards in the face-down pile compared to the face-up pile above it?

Do the trick a few times and write down the number of red and black cards in each pile. Share this data with other students. Is there a pattern in the numbers?

Task 2: Make a list of variables and constraints.

Variables are things that can change while constraints are things that are always fixed.

What about the number of face-up red cards dealt: does this vary each time the trick is performed?

What about the total number of black cards in all the piles: can that ever be different?

Task 3: Write these variables and constraints using letters instead of numbers. The number and colours of cards on the table can be represented as follows:

» **R0** (Red cards face up)
» **R1+B1** (Face-down card pile called the red pile)

» **B0** (Black cards face up)
» **R2+B2** (Face-down card pile called the black pile)

Can you create a set of equations from this?

What does R0 equal in terms of the other variables?

How can you use these equations and a simple equation substitution to help show that the trick always works?

What is a substitution? If, for example, A=B+C and C=2D, then we can replace C in the first equation with its value 2D to show that A=B+2D.

Extend

WHAT'S SIMILAR?

Task: You have created a mathematical model for the card trick and used mathematics to prove that it will always work: what other sorts of scientific or engineering processes would you want to be sure worked every time, even under different conditions? Aircraft landing gear? Buildings? Bridges?

Evaluate

WHAT'S MY UNDERSTANDING?

Task: What if you wanted to perform the trick differently so that, rather than having equal numbers of red and black cards in the face-down piles, the result was that there would be exactly one more red card than black cards?

What could you do?

Would it always work?